



HUSO Dubai Mathematics Final Round 9-10th Grade Exam

Student ID: _____

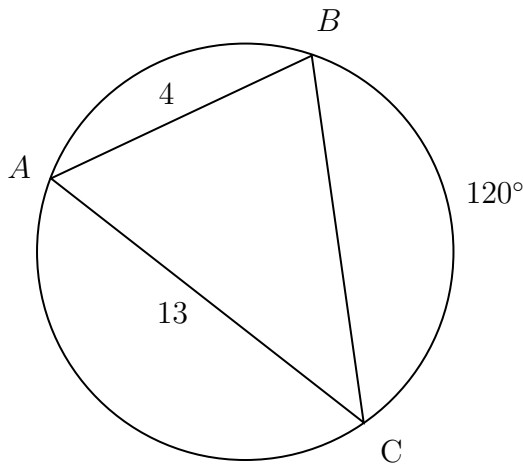
Name: _____

DO NOT OPEN THIS TEST BOOKLET UNTIL INSTRUCTED
Please read the following instructions carefully.

- This question booklet contains 25 questions, 6 sides (including the cover). If you discover any missing pages, please raise your hand.
- You have 60 minutes to complete this exam.
- You must bring a #2/HB pencil or a pen. Rulers, compass, and erasers are also allowed.
- Please bubble in your answers clearly and darkly on the provided answer sheet. Answers not bubbled in will NOT be graded.
- If you need to change an answer, please erase as clearly as possible. Unclear bubbles may be graded incorrectly.
- NO work in this booklet will be graded. You may use this booklet as extra scratch paper. Additional blank scratch paper will be available.
- NO CALCULATORS OR PHONES AND SIMILAR ELECTRONIC DEVICES OF ANY KIND ARE ALLOWED.
- Figures are not necessarily drawn to scale.
- This exam will be graded as follows:
 - Correct answer: +1 pts
 - Incorrect answer: -0.25 pts
 - Blank answer: 0 pts

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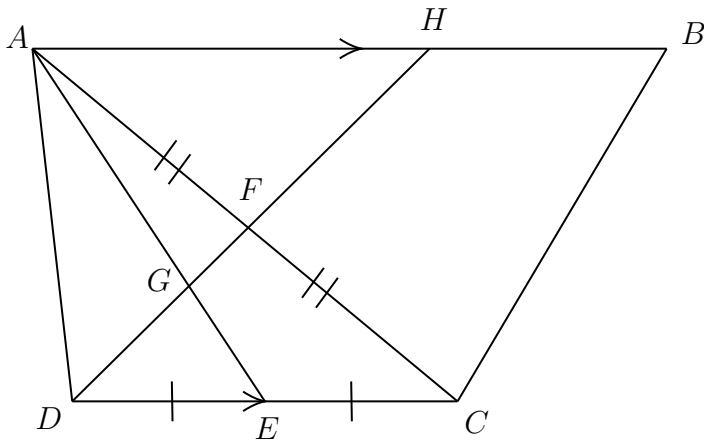
1. For how many non-empty subsets of $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ will product of all the elements of the subset be an even number?
A. 240 B. 364 C. 448 D. 496 E. 512
2. Let $S(x)$ return the sum of the digits in the base 6 representation of a base 10 number x . What value of x results in the largest value of $S(x)$ for $x < 1000$?
A. 825 B. 847 C. 863 D. 923 E. 999
3. $\triangle ABC$ is inscribed in a circle as below. $AB = 4$ and $AC = 13$ The arc between points B and C is 120° . What is the area of $\triangle ABC$?



- A. 13 B. $13\sqrt{3}$ C. 24 D. 26 E. $26\sqrt{3}$
4. Let n be the smallest number greater than 1000 such that $n \equiv 2 \pmod{5}$ and $n \equiv 3 \pmod{7}$. What is the sum of the digits of n ?
A. 6 B. 7 C. 8 D. 9 E. 10
5. Let $f(x) = (2x + 5)^3 - 18x - 45$. Let p be the largest root of $f(x)$ and q be the smallest root of $f(x)$. Find $p - q$.
A. 3 B. 4.5 C. 5 D. 6.5 E. 7
6. The line $y = ax + b$ is tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at $(-2, 0)$. What is ab ?
A. $\frac{32}{9}$ B. 4 C. $\frac{42}{9}$ D. $\frac{16}{3}$ E. 6
7. You are going to roll two fair dice. However, before each roll you flip a coin. If the coin lands heads, you roll a 6-sided die. If the coin lands tails, you roll an 8-sided die. What is the expected value of the sum of your two rolls?
A. 7 B. 8 C. 9 D. 10 E. 11

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8. Let n be the sum of the coefficients of $(x+3)^{10}$. What is the remainder when n is divided by 81?
- A. 0 B. 8 C. 14 D. 26 E. 31
9. Let $n = 2025^4 + 4 \cdot 2025^3 + 6 \cdot 2025^2 + 4 \cdot 2025 + 1$. How many factors does n have?
- A. 16 B. 25 C. 45 D. 64 E. 125
10. Alice is going to enter a lottery. There is a jar with balls labelled 1 to n , where n is equally likely to be any number from 2 to 2026. Two balls are drawn, one at a time, without replacement, to form the ordered pair (x, y) . The person who has been assigned this lucky ordered pair will win. Alice is assigned the lucky ordered pair $(1, 2)$. What is the probability that Alice wins the lottery?
- A. $\frac{1}{4052}$ B. $\frac{2025}{2026^2}$ C. $\frac{1}{2026}$ D. $\frac{2026}{2025^2}$ E. $\frac{2}{1013}$
11. In a geometric sequence, the last term is -768 , the common ratio is -2 , and the sum of all the terms is -516 . What is the third term in this sequence?
- A. -96 B. -24 C. -48 D. 24 E. 48
12. Quadrilateral $ABCD$ has area 30. Lines AB and DC are parallel. Line segment DH bisects AC at point F such that $AF = FC$. Line segment AE bisects DC such that $DE = EC$. Point G is the intersection of AE and DH . If triangle ADG has area 4, what is the area of quadrilateral $HBCF$?



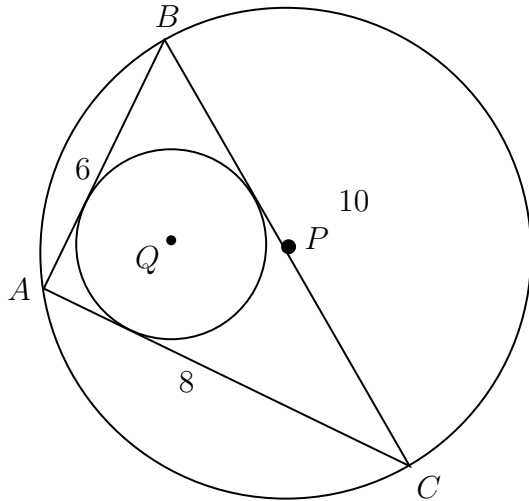
- A. 8 B. 12 C. 16 D. 20 E. 24
13. What is sum of all positive integers n such that $n \leq 2026$ and $(n+1)^{27} + (n+2)^{27} + (n+3)^{27}$ is divisible by 7?
- A. 291744 B. 292757 C. 294785 D. 300123 E. 303424

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14. Alice is eating jellybeans one-by-one from a jar with 4 red jellybeans, 3 green jellybeans, and 6 blue jellybeans. What is the probability that Alice eats the last green jellybean, then the last blue jellybean, then the last red jellybean?

A. $\frac{4}{39}$ B. $\frac{2}{13}$ C. $\frac{8}{39}$ D. $\frac{4}{13}$ E. $\frac{6}{13}$

15. Let $\triangle ABC$ have side lengths $AB = 6$, $AC = 8$, $BC = 10$. $\triangle ABC$ is inscribed in a circle centered at P as pictured below. Additionally, a smaller circle centered at Q is inscribed inside of $\triangle ABC$. What is the distance between P and Q ?



A. $\sqrt{5}$ B. $\sqrt{7}$ C. 3 D. $\sqrt{11}$ E. $2\sqrt{3}$

16. When $35!$ is computed, it ends in 8 zeros. What is the digit that immediately precedes these zeros?

A. 2 B. 4 C. 6 D. 8 E. 9

17. Let a, b, c, d be the roots of $f(x) = 3x^4 + 27x^2 + 3x - 2026$. What is the value of the following expression?

$$-1 * \frac{a^2 + 2ab + ac + ad + b^2 + bc + bd}{c^2 + ac + bc + 2cd + d^2 + ad + bd}$$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. 2

18. Suppose I have a rectangular prism with side lengths 2, 4, and 6. I chop off each corner of the prism such that the tips of the chopped off portion meet at the midpoint of each edge. What is the surface area to volume ratio of the resulting object?

A. $\frac{1}{3}$ B. $\frac{5}{9}$ C. $\frac{9}{5}$ D. $\frac{5}{2}$ E. 3

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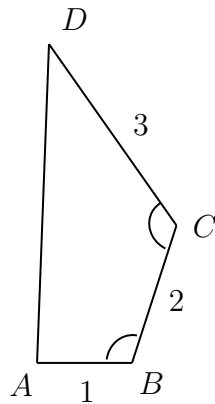
19. 6 (distinguishable) dogs and 6 (distinguishable) cats are sitting in a circle uniformly at random. What is the probability that exactly 5 dogs sit in a row somewhere in the circle?

- A. $\frac{3}{77}$ B. $\frac{5}{77}$ C. $\frac{1}{11}$ D. $\frac{9}{77}$ E. $\frac{11}{77}$

20. The numbers $\log(a^2b^3)$, $\log\left(\frac{b^{10.5}}{a}\right)$, $\log(a^8)$ are the first three numbers in an arithmetic sequence. The 18th element in this sequence can be written in the form $\log(b^x)$. What is x ?

- A. 45 B. 48 C. 54 D. 57 E. 60

21. Suppose that quadrilateral $ABCD$ has $AB = 1$, $BC = 2$, and $CD = 3$. $\angle ABC = \angle BCD = 120^\circ$. What is the length of side AD ?



- A. $\sqrt{17}$ B. $\sqrt{19}$ C. $\sqrt{21}$ D. $2\sqrt{6}$ E. $3\sqrt{3}$

22. Suppose 8 points are chosen on the perimeter of a circle uniformly at random. What is the probability that all 8 points lie on the same half of the circle?

- A. $\frac{1}{32}$ B. $\frac{1}{16}$ C. $\frac{3}{32}$ D. $\frac{1}{8}$ E. $\frac{3}{16}$

23. Let n be a value such that $260n^2$ has exactly 260 positive factors, including 1 and $260n^2$. What is the maximum number of factors that $125n$ can have?

- A. 128 B. 164 C. 192 D. 256 E. 312

24. Alice and Bob each pick integers between 0 and 100 (inclusive), uniformly at random and independently of each other. Alice doesn't know her own number, but she did see that Bob's number has a 4. The probability that Alice has a strictly larger number than Bob can be expressed as a fraction $\frac{a}{b}$ in lowest terms. What is $b - a$?

- A. 191 B. 347 C. 562 D. 839 E. 910

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25. Consider the following system of equations:

$$\begin{aligned}x^2y^2 - x^2 - y^2 &= 6 \\x^2z^2 - x^2 - z^2 &= 0 \\3y^2z^2 - 3y^2 - 3z^2 &= 18\end{aligned}$$

What is the value of y^2 ?

- A. 4 B. 6 C. 8 D. 10 E. 12